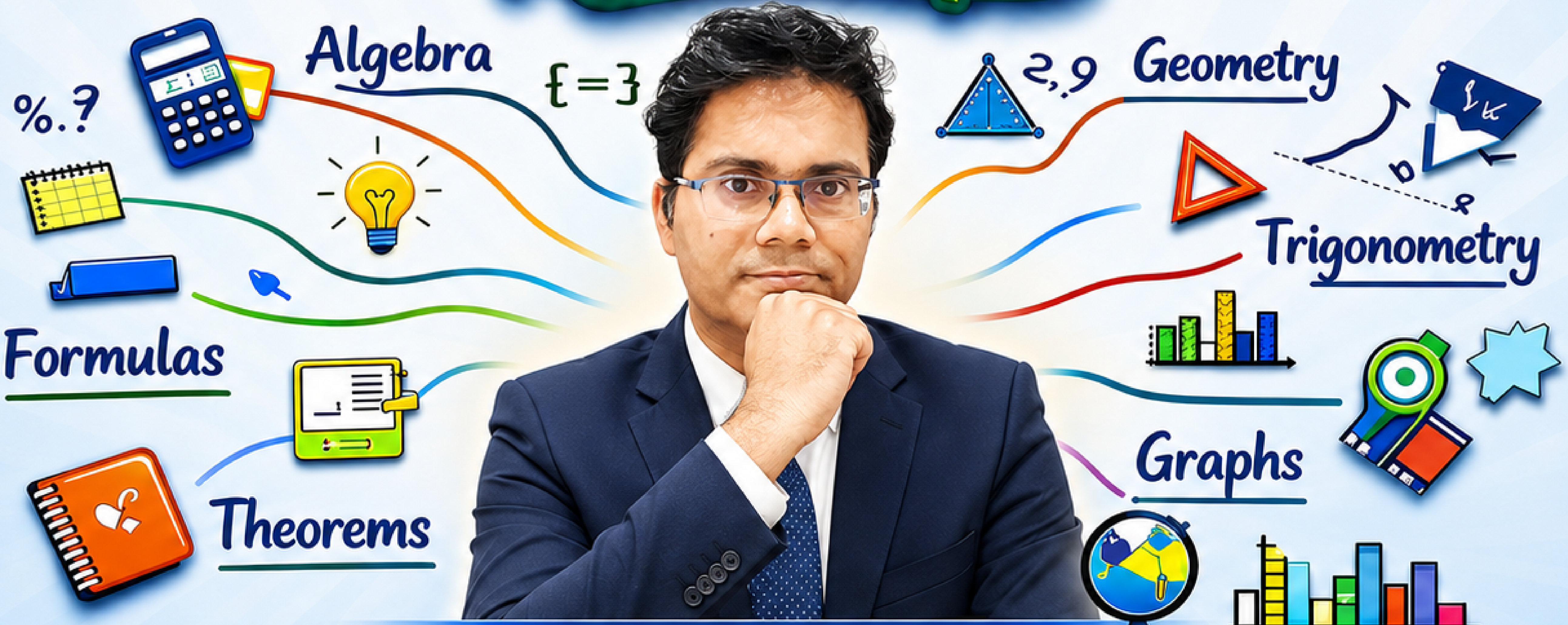
From the author of **Mathmission** series books

MATHEMATICS
CLASS 10
Mind Maps



O.P. GUPTA

Indira Award Winner

MIND MAPS
Class X - Maths

REAL NUMBERS

Type of Numbers (Number System)

- Natural numbers (**N**) : 1, 2, 3, ...
- Whole numbers (**W**) : 0, 1, 2, 3, ...
- Integers (**Z**) : $0, \pm 1, \pm 2, \pm 3, \ldots$
- Rational numbers (**Q**) : $\frac{7}{5}, -\frac{1}{4}, \frac{13}{19}, 9$ i.e., $\frac{9}{1}, \frac{16}{3}, \ldots$
- Irrational numbers (**T**) : $\sqrt{3}, 1-\sqrt{7}, 4\sqrt{2}, 2+3\sqrt{5}, \frac{\sqrt{11}}{5}$, e, 0.1212212221..., 0.030030003..., etc.
- Real numbers (**R**) : $-3, 0, 1, 2, -\frac{1}{5}, \frac{6}{11}, \sqrt{7}$ etc.

Prime numbers

Prime numbers : A prime number is a natural number greater than 1 that has exactly two distinct positive divisors, 1 and the number itself.
In other words, we can say a prime number can be divided only by 1 and itself, not by any other number.

- 2 is the only even prime number.
- 1 is NOT a prime number.

Coprime numbers : Two numbers are said to be coprime (or relatively prime) if their HCF is 1. Coprime numbers do not have any common factor other than 1.

Fundamental Theorem of Arithmetic (FTA)

Every composite number can be expressed (factorized) uniquely as a product of prime numbers except for order.

☑ Applications :
- Prime factorization
- Finding HCF and LCM

Revisiting Irrational Numbers

Def. Irrational numbers cannot be expressed as a ratio of two integers i.e., they cannot be written in the form of $\frac{p}{q}, q \neq 0$. Irrational numbers are non-terminating and have non-repeating decimal expansion.

Examples: $\sqrt{2}, \sqrt{3}, 5-2\sqrt{7}, \pi$ etc.

- **Terminating rational numbers :** $\frac{1}{2}$ i.e., 0.5, $\frac{71}{100}$ i.e., 0.71 etc.
- **Non-terminating rational numbers with the repeating decimal expansion :** $\frac{1}{3}$ i.e., 0.333..., 0.4232323... etc.
- **Irrational numbers with Non-Terminating and Non-Repeating Decimal expansion :** 0.101001000100001..., 1.1101001000100001... etc.

Applications of HCF and LCM

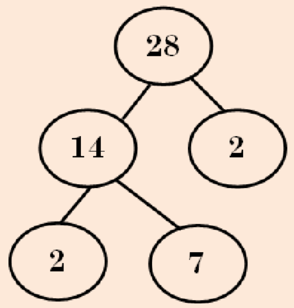
- HCF : Greatest number dividing two or more numbers
- LCM : Smallest number divisible by two or more numbers
- Relationship : HCF $(a, b) \times$ LCM $(a, b) = a \times b$
- Real life application based word problems on grouping or periodic events, time

Proof of irrationality

- Prove that $\sqrt{3}$ is irrational.
- Prove that $3-5\sqrt{11}$ is irrational.
- If $\sqrt{2}$ is given as an irrational number, then prove that $(5-2\sqrt{2})$ is an irrational number.

- HCF = Product of common prime factors with lowest exponents
- LCM = Product of all prime factors with highest exponents
- HCF is the divisor of LCM.
- HCF of two coprime numbers is always 1.
- LCM of two coprime numbers is equal to the product of those two numbers.

Factor Tree



Def. A factor tree is a visual method of representing the prime factorization of a number by expressing it as a product of prime numbers. Refer the adjacent figure which illustrates the prime factorization of 28.

Clearly, $28 = 2 \times 2 \times 7$

Check whether a^n can end with the digit 0, where 'a' and 'n' are any natural numbers?

- For a^n to end with the digit 0, the prime factorization of 'a' must include '2' and '5' both.

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MIND MAPS
Class X - Maths

POLYNOMIALS

General form of a polynomial

A polynomial is defined as an algebraic expression of the following from

$$\mathbf{p(x) = a_0 + a_1x^1 + a_2x^2 + a_3x^3 + \ldots + a_nx^n,\ a_n \neq 0},$$

where all the exponents (i.e., powers) are non-negative integers and $a_0, a_1, a_2, \ldots, a_n$ are real nos. (s.t. $a_n \neq 0$).

Type of Polynomials & their standard forms

- Constant : $p(x) = c$, where c is any non-zero real no.
- Zero : $p(x) = 0$ (sometimes treated as a **special case** of constant polynomial)
- Linear : $p(x) = ax + b,\ a \neq 0$
- Quadratic : $p(x) = ax^2 + bx + c,\ a \neq 0$
- Cubic : $p(x) = ax^3 + bx^2 + cx + d,\ a \neq 0$
- Biquadratic : $p(x) = ax^4 + bx^3 + cx^2 + dx + e,\ a \neq 0$
- Quintic : $p(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f,\ a \neq 0$

For a quadratic polynomial, $p(x) = ax^2 + bx + c,\ a \neq 0$

- **Sum of zeroes (S) :** $-\frac{b}{a}$
- **Product of zeroes (P) :** $\frac{c}{a}$

Forming a quadratic polynomial

$x^2 - Sx + P$ or $k(x^2 - Sx + P)$ where k is any non-zero real number.

Note that, S : Sum of zeroes of quadratic polynomial and P : Product of zeroes of quadratic polynomial

Nature of biquadratic polynomial
Degree : 4
Graph : W-shaped or U-shaped
No. of zeroes : Four zeroes (at most four *real* zeroes), cuts x-axis at four points on graph in the case of *all* real zeroes

Nature of cubic polynomial
Degree : 3
Graph : Inverted S-shaped
No. of zeroes : Three zeroes (at most three *real* zeroes), cuts x-axis thrice on graph in the case of *all* real zeroes

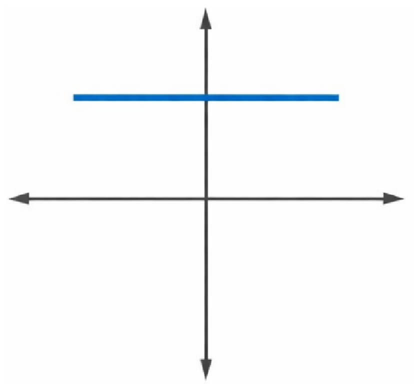
Nature of quadratic polynomial
Degree : 2
Graph : Parabolic
No. of zeroes : Two zeroes (at most two *real* zeroes), cuts x-axis twice on graph in the case of *all* real zeroes

Nature of linear polynomial
Degree : 1
Graph : Straight line
No. of zeroes : One *real* zero, cuts x-axis once on graph

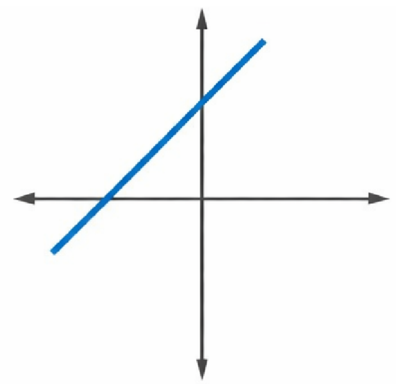
Nature of zero polynomial
Degree : Not defined
Graph : Coincident on x-axis (i.e., x-axis itself)
No. of zeroes : Infinitely many zeroes

Nature of constant polynomial
Degree : 0
Graph : Straight line parallel to x-axis
No. of zeroes : No zeroes (0 count of zeroes)

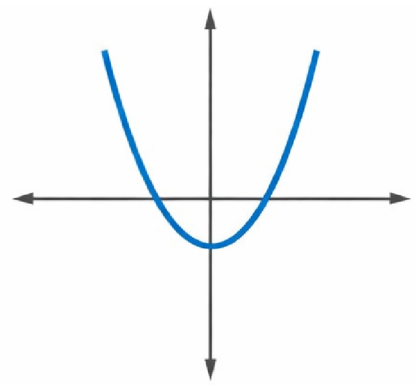
Constant polynomial, degree : 0



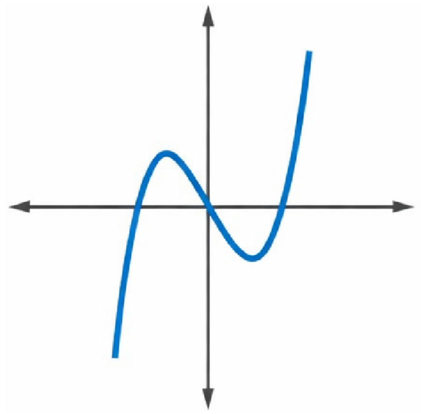
Linear polynomial, degree : 1



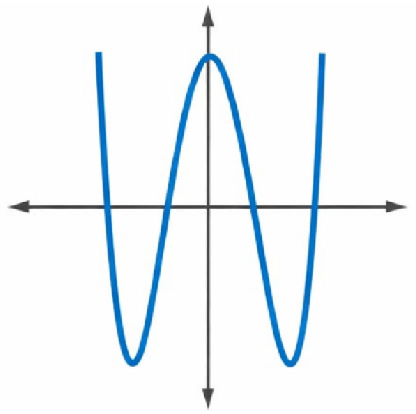
Quadratic polynomial, degree : 2



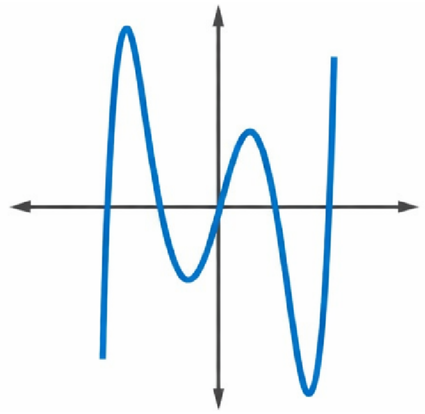
Cubic polynomial, degree : 3



Biquadratic polynomial, degree : 4



Quintic polynomial, degree : 5



Algebraic Identities

- $(a \pm b)^2 = a^2 \pm 2ab + b^2$
- $a^2 - b^2 = (a+b)(a-b)$
- $(a \pm b)^3 = a^3 \pm 3ab(a \pm b) \pm b^3$

Note that, $a^2 + b^2 = (a+b)^2 - 2ab$, $(a+b)^4 = [(a+b)^2]^2$, $(a+b)^6 = [(a+b)^3]^2$ etc.

Why do we say "Equal roots" not "One root" for some equations corresponding to some polynomials?

Consider a polynomial, $p(x)=x^2-10x+25$.
Here $p(x)=x^2-10x+25=0$ has "Equal roots" not "One root".
Using the factorization gives us the same value i.e., $p(x)=(x-5)(x-5)=0$ implies, $x=5$.
But why do we say it has "**one repeated root**" or "**equal roots**" i.e., indicating we have **two roots but with the same value**.
Why not say it has one root only?

This is due to the **Fundamental Theorem of Algebra**.

"Every polynomial of order n has **exactly** n zeroes."
That is, each equation corresponding to the polynomial of degree n has **exactly** n roots.

A polynomial is an expression with non-negative integer powers of x i.e., $a+bx+cx^2+dx^3+\ldots$.
All linear, quadratic and cubic expressions are the examples of polynomials.

The order of a polynomial is its highest power of x. The order of a quadratic is 2, a cubic 3 and so on.

These roots might be repeated or might not be **'real'** roots.
The root of polynomial may be an imaginary number. But it is still a value!

The **fundamental theorem of algebra** means that a quadratic equation $ax^2+bx+c=0$ (order 2) will **always** have 2 roots. This is why we say **no real roots** when $b^2-4ac<0$ rather than **no roots**. Similarly, we must say **equal roots** when $b^2-4ac=0$. (We shall study about the quantity b^2-4ac and **nature of roots** for a quadratic equation in Chapter-4).

Despite the theorem being a simple statement, it was only until 1806 that it was first proven by Jean-Robert Argand. Clearly by using the quadratic formula we can show a quadratic equation has 2 roots (corresponding to a quadratic polynomial having 2 zeroes). We can use similar formulae to show that a cubic equation has 3 roots and a quartic equation has 4 roots. Curiously, there is provably no such formulae for order 5 (quintics) and beyond. Therefore to prove the theorem we must prove for example that 5 roots exist for a quintic, despite us having no way to find these exact roots!
One side result of the Fundamental Theorem of Algebra is that every polynomial can be written as a **product of linear and/or quadratic expressions**.
Leibniz claimed in 1702 that a polynomial of the form x^4+a^4 cannot be written in this way. He then got totally burned by Euler in 1742 who managed to do so, $x^4+a^4=(x^2+a\sqrt{2}x+a^2)(x^2-a\sqrt{2}x+a^2)$.

MIND MAPS
Class X - Maths

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Def. A *pair of linear equations in two variables* is a set of **two equations**, each of **degree 1**, involving the **same two variables** (usually x and y).

✪ General form : $\mathbf{a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0}$, where $a_1, b_1, a_2, b_2 \neq 0$ and $a_1, b_1, c_1, a_2, b_2, c_2$ are real nos.

How to solve the given system of linear equations in two variables?

Graphical method

In this method, we draw the graphs of both equations on the same Cartesian plane. The **point of intersection** gives the solution (x, y).

Important points to note :

- Intersecting lines → one solution
- Coincident lines → infinitely many solutions
- Parallel lines → no solution

Algebraic method

I. Substitution method

1. Express one variable in terms of the other from one equation
2. Substitute this expression (obtained above) into the second equation
3. Solve for one variable
4. Substitute the obtained value of one variable back into any one equation to find the value of other variable

II. Elimination method

1. Make the coefficients of one variable equal in both equations
2. Add or subtract the equations to eliminate one variable
3. Solve the resulting equation to get value of one variable
4. Substitute the obtained value of one variable into any one equation to get the value of second variable

Word Problems (Applications based problems)

We normally find problems based on

- Numbers
- Ages
- Geometry
- Money
- Speed, distance & time

Conditions of Consistency and inconsistency of the pair of lines / Graphical representation

Pair of Equations	Comparing the Ratio	Graphical meaning	Algebraic Interpretations
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting Lines	Unique Solution (Consistent system)
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident Lines	Infinitely Many Solutions (Consistent system)
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel Lines	No Solution (Inconsistent system)

Special case when Coefficient of x and y are interchanged in the two equations

Consider the two equations as $ax + by = m$ and $bx + ay = n$.

Now do as per the following steps.

1. Add the two equations to obtain $(a+b)x + (b+a)y = m + n$
   $\Rightarrow (a+b)(x+y) = m+n$
   $\Rightarrow x + y = \dfrac{m+n}{a+b}$ ...(i)
2. Subtract the second equation from the first equation to obtain $(a-b)x + (b-a)y = m - n$
   $\Rightarrow (a-b)(x-y) = m-n$
   $\Rightarrow x - y = \dfrac{m-n}{a-b}$ ...(ii)
3. Add (i) and (ii) to obtain the value of x and then subtract (ii) from (i) to obtain value of y.

QUADRATIC EQUATIONS

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General form of a Quadratic equation

An equation of the form $\mathbf{ax^2+bx+c=0}$, where a, b and c are real numbers such that $a \neq 0$ is called a quadratic equation.

Solutions (roots) of a quadratic equation

Since $ax^2+bx+c=0$ is of degree 2, so it shall have two roots (including real and/or imaginary roots).
Let $x=\alpha$ and $x=\beta$ be the roots of given quadratic equation $ax^2+bx+c=0$, $a \neq 0$. Then we shall have $a\alpha^2+b\alpha+c=0$ and $a\beta^2+b\beta+c=0$.

Discriminant and nature of roots of quadratic equation $\mathbf{ax^2+bx+c=0}$, $\mathbf{a \neq 0}$

We calculate b^2-4ac, which is called Discriminant (D).

The discriminant (D) helps us in deciding the **nature of roots** of the quadratic equation without actually solving it. Thus a quadratic equation $ax^2+bx+c=0$ has

- *real solutions if* $D \geq 0$.
- *two distinct real roots if* $D > 0$.
- *two equal and real roots if* $D = 0$.
- *no real roots if* $D < 0$.

Note that, the equal roots of quadratic equation are also known as the **Coincident roots**.

For a quadratic equation, $\mathbf{ax^2+bx+c=0,\ a \neq 0}$

- **Sum of roots (S) :** $-\dfrac{b}{a}$
- **Product of roots (P) :** $\dfrac{c}{a}$

Forming a quadratic equation

$x^2-Sx+P=0$ or $k(x^2-Sx+P)=0$ where k is any non-zero real number.

Note that, S : Sum of roots of quadratic equation and P : Product of roots of quadratic equation

Word Problems (Applications based problems)

We normally find problems based on

- Numbers
- Ages
- Geometry
- Speed, distance & time

Methods of solving quadratic equations

(a) Factorization method:

Firstly factorize $ax^2+bx+c=0, a \neq 0$ into a product of two linear factors *say* $(x-\alpha)(x-\beta)=0$ by splitting the middle term. Then equate each factor to zero and get the values of x as $x=\alpha$ and $x=\beta$. These values of x (i.e., α and β) are called **roots of the given quadratic equation**.

(b) Quadratic formula method or Discriminant method:

Obtain the discriminant (D) of quadratic equation $ax^2+bx+c=0, a \neq 0$ that is, $D=b^2-4ac$. Then the roots of the given quadratic equation are $\dfrac{-b+\sqrt{D}}{2a}$ and $\dfrac{-b-\sqrt{D}}{2a}$ $\left(\text{i.e., } x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}\right)$.

The Quadratic formula method is also called **Sridharacharya method**, after the name of famous Indian mathematician Sridhara (or, Sridharacharya) who derived this formula.

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ARITHMETIC PROGRESSION

Def. of Arithmetic Progression

A succession of numbers is said to be in A.P. if the difference between any term and the term preceding it is constant throughout. That is, an A.P. is a sequence in which the difference between consecutive terms is constant. This constant is called as the **Common difference** of the arithmetic progression (A.P.) and is represented by the lower case letter **'d'**.

Thus, if $a_1, a_2, a_3, ..., a_n$ represent the n terms of an A.P., then $a_2 - a_1 = a_3 - a_2 = ... = a_n - a_{n-1}$. Also, the number obtained by $a_2 - a_1$ or $a_3 - a_2$ i.e., $a_n - a_{n-1}$ is called the common difference (denoted by 'd').

Thus, $d = a_2 - a_1$, and in general $d = a_n - a_{n-1}$.

Clearly, the general form of an A.P. can be written as $a, a+d, a+2d, a+3d, ...$.

- a : first term, d : common difference of A.P.

General term i.e., n^{th} term of A.P.

If an A.P. has n terms with first term 'a' and the common difference 'd', then its n^{th} term is given as $a_n = a + (n-1)d$. Remember that a_n is called the **last term** and **n^{th} term** of the A.P. as well.

Important points to note :

- Expression involving n^{th} term of an A.P. is always **linear**.

p^{th} term of an A.P. from the end

Consider an A.P. having n terms with first term and common difference as 'a' and 'd' respectively. Then, p^{th} term from end $= (n-p+1)^{th}$ term from beginning.

Hence, $a_{n-p+1} = a + ((n-p+1)-1)d = a + (n-p)d$.

Further, p^{th} term from end $= a_n + (p-1)(-d)$.

Also we can **use the following algorithm** too to find the p^{th} term of an A.P. from end.

STEP 1 Consider the given A.P. as $a_1, a_2, a_3, ..., a_n$.

STEP 2 Reverse the A.P. to obtain a new A.P. as $a_n, a_{n-1}, a_{n-2}, ..., a_2, a_1$.

STEP 3 Now consider a_n as the first term of this newly obtained A.P. with $d = a_{n-1} - a_n$.

STEP 4 Find p^{th} term from the beginning of this new A.P. This will give the p^{th} term from the end for the original arithmetic progression.

Properties of A.P.

(a) If the same number is added to, or subtracted from, all the terms of an A.P. then the resulting progression is also an A.P. with the **same common difference** as that of the original A.P.

(b) If the corresponding terms of two Arithmetic progressions be added or subtracted, the resulting progression is also an A.P. with common difference $d_1 + d_2$ or $d_1 - d_2$ (or, $d_2 - d_1$) as the case may be of addition or subtraction of arithmetic progressions respectively.

(c) If all the terms of an A.P. be multiplied or divided by the same quantity (non-zero number), the resulting progression is also an A.P. with common difference kd or $\frac{d}{k}$ as the case may be of multiplication or division respectively.

- If a, b, c are in A.P., then $2b = a + c$ i.e., $b = \frac{a+c}{2}$. Also the number 'b' is called the **Arithmetic Mean** (A.M.) between a and c.
- Any three numbers in A.P. can be considered as: $a-d, a, a+d$.
- Any four numbers in A.P. can be considered as: $a-3d, a-d, a+d, a+3d$.
- Any five numbers in A.P. can be considered as: $a-2d, a-d, a, a+d, a+2d$.

Sum of n terms of A.P. (S_n)

Consider $a_1, a_2, a_3, ..., a_n$ represent an A.P. with n terms. Let $S_n = a_1 + a_2 + a_3 + ... + a_n$. Then the sum of n terms of A.P. will be given by

- $S_n = \frac{n}{2}[2a + (n-1)d]$
- $S_n = \frac{n}{2}(a + a_n) = \frac{n}{2}(\text{first term} + \text{last term})$

Important points to note :

- Expression involving sum of n terms of an A.P. is always **quadratic**.
- $\mathbf{S_1 = a_1}$
- $\mathbf{a_n = S_n - S_{n-1}}$
- $\mathbf{S_n = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}}$ **= Sum of first n natural numbers**

MIND MAPS
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TRIANGLES

Pythagoras Theorem
In a right-angled triangle (i.e., one angle is of 90°), we always have $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$.

Converse of Pythagoras Theorem
If $(\text{Longest side})^2 = $ Sum of squares of other two sides, then the triangle is right-angled.

Ratio of Corresponding Sides in Similar Triangles
If two triangles are **similar**, then **all their corresponding linear dimensions are in the same ratio**. That is, in two similar triangles, the **ratio of corresponding sides** is **equal** to the ratio of their
- corresponding medians
- corresponding altitudes
- corresponding angle bisectors

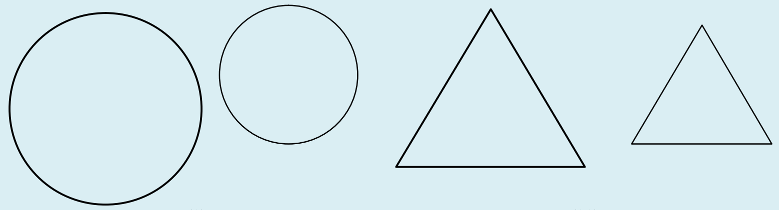
Explanation :

Let $\Delta ABC \sim \Delta DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

1. **Ratio of medians :** If AM and DN are medians of $\triangle ABC$ and $\triangle DEF$ respectively, then $\frac{AM}{DN} = \frac{AB}{DE}$.
2. **Ratio of altitudes :** If AL and DP are altitudes of $\triangle ABC$ and $\triangle DEF$ respectively, then $\frac{AL}{DP} = \frac{AB}{DE}$.
3. **Ratio of angle bisectors :** If AQ and DR are angle bisectors of corresponding angles in $\triangle ABC$ and $\triangle DEF$ respectively, then $\frac{AQ}{DR} = \frac{AB}{DE}$.

Note : Above results can be used directly to answer the MCQs.

Similar figures
If two figures are given, having the same shape but *not necessarily* the same size, then they are called the similar figures. Refer the following figures.



(i) (ii)

- All the congruent figures are similar but the converse is not necessarily true.
- Two polygons of the same number of sides are similar, if
  (i) their corresponding angles are equal and
  (ii) their corresponding sides are in same ratio.

☑ Thales Theorem i.e., Basic Proportionality Theorem (B. P. T.)
If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

✪ Proof is important.

☑ Converse of B. P. T. i.e., Converse of Thales Theorem
If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

✪ Result is important (proof is not important).

Important Applications
- Finding unknown sides
- Verifying similarity
- Finding heights and distances
- Solving real-life problems

Common mistakes to avoid
- Mixing congruence and similarity
- Incorrect correspondence of sides

Similar triangles
Two triangles are said to be similar, if
- their **corresponding angles are equal**
- their **corresponding sides are proportional** i.e., the **corresponding sides are in the same ratio**.

Criterion for Similarity of triangles
(a) A. A. A. Similarity (Angle-Angle-Angle) or A. A. Similarity (Angle-Angle)

If all three angles of one triangle are equal to the three angles of another triangle, then the triangles are similar i.e., $\Delta ABC \sim \Delta PQR$, when $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$.

Or, A. A. Similarity (Angle-Angle)

If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar i.e., $\Delta ABC \sim \Delta PQR$, when $\angle A = \angle P$ and $\angle B = \angle Q$.

Note : If two angles in both triangles are same, then the third angle will also be same.

(b) S. A. S. Similarity (Side-Angle-Side)

If the ratio of two sides is equal and included angle is equal, then the triangles are similar i.e., $\Delta ABC \sim \Delta PQR$, when we have $\angle A = \angle P$ and $\frac{AB}{PQ} = \frac{AC}{PR}$.

(c) S. S. S. Similarity (Side-Side-Side)

If the corresponding sides are proportional, then the triangles are similar i.e., $\Delta ABC \sim \Delta PQR$, when we have $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$.

MIND MAPS
Class X - Maths

COORDINATE GEOMETRY

Cartesian Coordinate System

- **X-axis** (horizontal)
- **Y-axis** (vertical)
- Both axes intersect at **Origin (0, 0)**

Coordinates of a Point is written as (x, y)

- $x \rightarrow$ Abscissa
- $y \rightarrow$ Ordinate

✪ Y coordinate of any point on x-axis is 0 i.e., (x, 0).
✪ X coordinate of any point on y-axis is 0 i.e., (0, y).

Quadrants and sign conventions

Quadrant	Sign of x	Sign of y	Sign of (x, y)
Quadrant I	Positive	Positive	(+, +)
Quadrant II	Negative	Positive	(–, +)
Quadrant III	Negative	Negative	(–, –)
Quadrant IV	Positive	Negative	(+, –)

Some useful facts

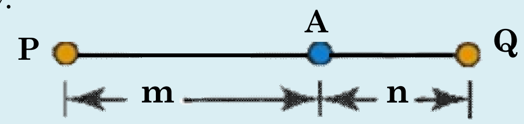
In order to prove that a given figure is

(a) an isosceles triangle, prove that its two sides are equal.
(b) an equilateral triangle, prove that all of its three sides are equal.
(c) a square, prove that the four sides are equal and the diagonals are also equal.
(d) a rhombus, prove that the four sides are equal.
(e) a rectangle, prove that the opposite sides are equal and the diagonals are also equal.
(f) a parallelogram, prove that the opposite sides are equal.
(g) a parallelogram but not a rectangle, prove that its opposite sides are equal but diagonals are not equal.
(h) a rhombus but not a square, prove that its all sides are equal but the diagonals are not equal.

Section Formula

Consider a line segment PQ such that their coordinates are $P(x_1, y_1)$ and $Q(x_2, y_2)$.

Also assume that a point $A(x, y)$ which lies somewhere in between PQ, then A must divide PQ in some ratio $m : n$ *say*.



PA : QA = m : n
(Internal division)

Then $A(x, y) = A\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$.

Note : If A is the **midpoint** of line-segment PQ, that implies A divides PQ in 1 : 1, then $A\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Distance Formula

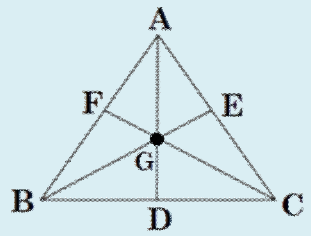
If points $A(x_1, y_1)$ and $B(x_2, y_2)$, then the distance between the points A and B is, $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Some Special Cases

- Distance of (x, y) from x-axis $= |y|$
- Distance of (x, y) from y-axis $= |x|$
- Distance of (x, y) from origin $= \sqrt{x^2 + y^2}$

Centroid (Centre of gravity)

It is the point of intersection of the medians of triangle. Let the coordinates of the ΔABC be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Then in order to locate the coordinates of the centroid, we use **section formula** with the fact that medians AD, BE and CF intersect at G (*say*) each other in the ratio $2 : 1$. Hence, $G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ is the centroid of ΔABC.



To prove three points as collinear

By collinear points we mean that they lie on the same line. Suppose three points under consideration are given as $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Then in order to prove that they are collinear we can do any of the followings.

(a) Prove that the sum of the distances between two pairs of points is equal to the third pair of points. That is $AB + BC = AC$ or, $AC + BC = AB$ or, $AC + AB = BC$.

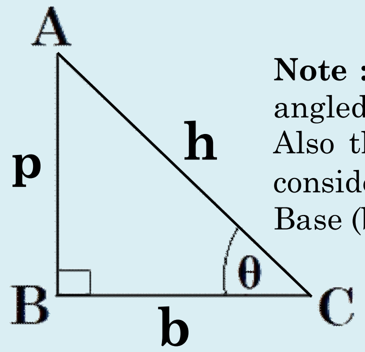
(b) If B lies on AC, then it must divide AC in some ratio which can be found by the help of section formula. Similarly in cases of other points it can be checked if the point lies on the join of remaining two points.

MIND MAPS
Class X - Maths

INTRODUCTION TO TRIGONOMETRY

Understanding a right angled triangle

Let ΔABC be a right angled triangle with $\angle B = 90^\circ$. Let $\angle ACB = \theta$ (an acute angle); AB = Perpendicular (p), BC = Base (b) and, CA = Hypotenuse (h).



Note : The longest side in the right angled triangle is Hypotenuse (h). Also the angle θ (i.e., the angle in consideration) is always between the Base (b) and Hypotenuse (h).

Trigonometric ratios

Refer the ΔABC shown adjacent.

- $\sin\theta = \frac{AB}{CA} = \frac{p}{h}$
- $\cos\theta = \frac{BC}{CA} = \frac{b}{h}$
- $\tan\theta = \frac{AB}{BC} = \frac{p}{b}$
- $\text{cosec}\,\theta = \frac{CA}{AB} = \frac{h}{p}$
- $\sec\theta = \frac{CA}{BC} = \frac{h}{b}$
- $\cot\theta = \frac{BC}{AB} = \frac{b}{p}$

✪ Note the following abbreviations which we prefer to use.

- Sine → sin
- Cosine → cos
- Tangent → tan
- Cosecant → cosec
- Secant → sec
- Cotangent → cot

Trigonometric ratios of Standard angles

Angles / T-Ratios	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Pythagoras Theorem

In a right-angled triangle (i.e., one angle is of 90°), we always have $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$.

That is, $h^2 = b^2 + p^2$ for ΔABC shown above.

Relationship between trigonometric ratios

(a) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

(b) $\sec\theta = \frac{1}{\cos\theta}$

(c) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

(d) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(e) $\tan\theta = \frac{1}{\cot\theta}$ or, $\cot\theta = \frac{1}{\tan\theta}$

Behavior of trigonometric ratios in a right angled triangle

- As angle θ increases from 0° to 90°, the value of $\sin\theta$, $\tan\theta$ and $\sec\theta$ increases.
- As angle θ increases from 0° to 90°, the value of $\cos\theta$, $\cot\theta$ and $\text{cosec}\,\theta$ decreases.

Trigonometric Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta = 1 + \tan^2\theta$
- $\text{cosec}^2\theta = 1 + \cot^2\theta$

Common mistakes to avoid

- Mixing up sin, cos, and tan ratios
- Using Pythagoras identity incorrectly
- Not identifying the right angle in triangle problems
- Forgetting reciprocal identities, e.g. $\sec\theta = \frac{1}{\cos\theta}$
- Wrong values of trigonometric ratios for standard angles
- Not writing the angles with trigonometric ratios, e.g. $\sin^2 + \cos^2 = 1$.

Trigonometric Ratios of Complementary Angles

- $\sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta$
- $\tan(90^\circ - \theta) = \cot\theta$ and $\cot(90^\circ - \theta) = \tan\theta$
- $\sec(90^\circ - \theta) = \text{cosec}\,\theta$ and $\text{cosec}(90^\circ - \theta) = \sec\theta$

Note : This concept is **not** in syllabus for 2025-26, however if required, you may use these to answer the MCQs.

MIND MAPS
Class X - Maths

SOME APPLICATIONS OF TRIGONOMETRY

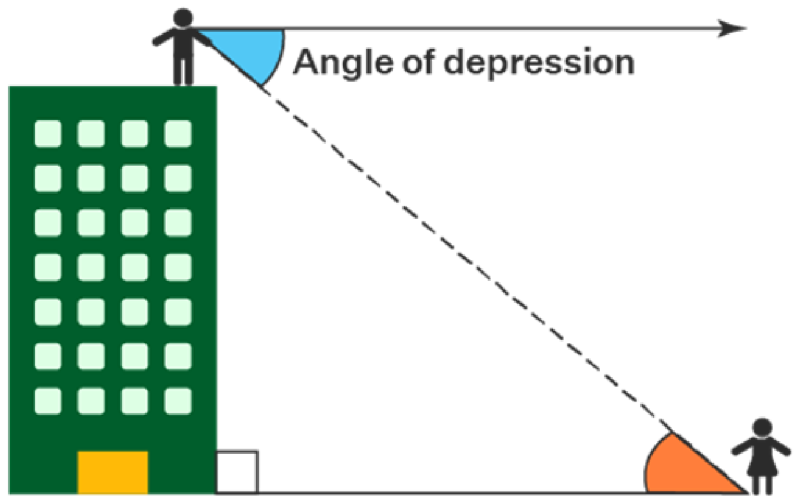
Right-angled triangle formation
- Horizontal line through the observer's eye
- Vertical object (tower, tree, building, hill)
- Perpendicular dropped from the object to the ground

Most commonly used trigonometric ratios
- $\tan\theta = \dfrac{\text{Height}}{\text{Distance}}$
- $\sin\theta = \dfrac{\text{Height}}{\text{Slant distance}}$
- $\cos\theta = \dfrac{\text{Distance}}{\text{Slant distance}}$

Common mistakes to avoid
- Confusing elevation and depression
- Ignoring height of observer
- Using wrong trigonometric ratio
- Missing units in final answer

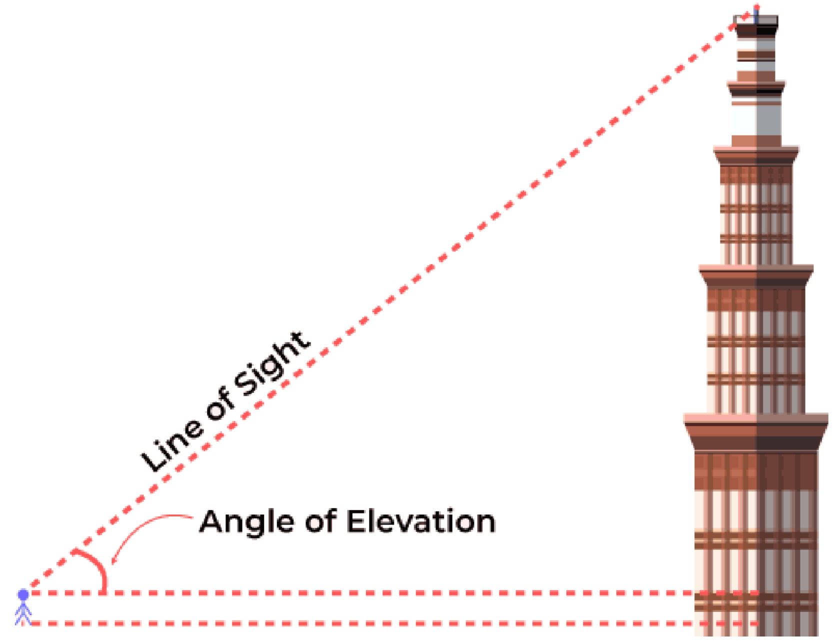
Line of Sight
- The line joining the **observer's eye** and the **object**
- Forms an angle with the horizontal

Angle of Depression
- Angle between the **line of sight** and the **horizontal**, when the object is **below** eye level
- For example: Looking down from the top of a building



Important tips for the board exams
- Draw a neat and labelled diagram
- Approximation only if required
- Use standard trigonometric values only
- Memorize $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$

Angle of Elevation
- Angle between the **line of sight** and the **horizontal**, when the object is **above** eye level
- For example: Looking at the top of a tower



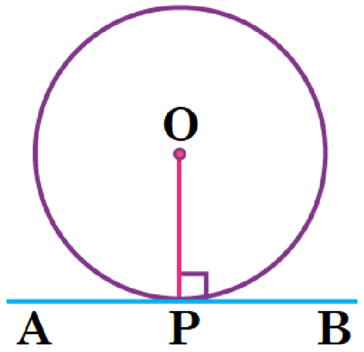
Steps to solve problems
1. Draw a **neat labelled diagram**
2. Identify angle (elevation / depression)
3. Choose suitable **trigonometric ratio**
4. Substitute correct values of trigonometric ratios
5. Solve and write **units**

MIND MAPS
Class X - Maths

CIRCLES

Theorems on Tangents

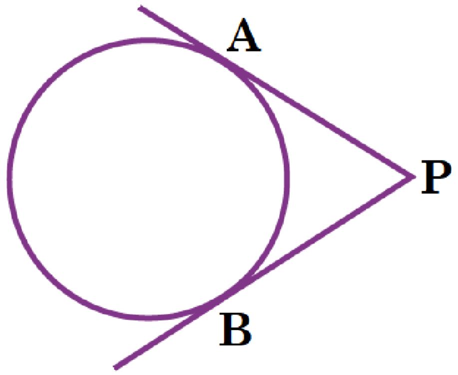
Theorem 1
The tangent at any point of a circle is perpendicular to the radius through the point of contact.



That is, $OP \perp AB$

✪ Proof is important.

Theorem 2
The lengths of tangents drawn from an external point to a circle are equal.



That is, $PA = PB$

✪ Proof is important.

Basic introduction

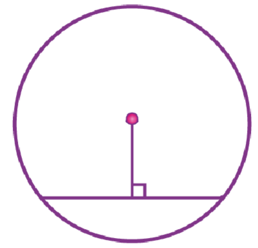
- A **circle** is the set of all points in a plane at a fixed distance from a fixed point
- Fixed point → **Centre**
- Fixed distance → **Radius**
- **Radius (r)** : Distance from centre to any point on the circle
- **Diameter (2r)** : Longest chord of the circle, passes through centre
- **Chord** : Line segment joining any two points on the circle
- **Arc** : Part of the circumference
- **Circumference** : Perimeter of the circle
- **Secant** : A line intersecting the circle at **two points**
- **Tangent** : A line which touches the circle at **exactly one point**, this point is called **Point of contact**

Important tips for exams

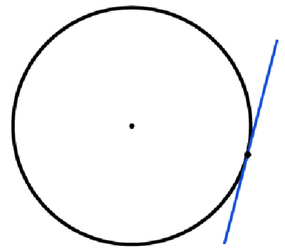
- Draw neat, labelled diagrams
- Mention theorem in problems
- Identify the point of contact accurately

Important facts

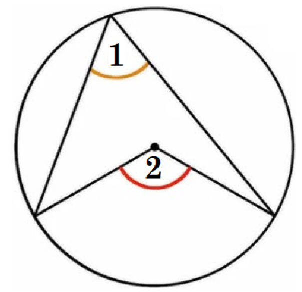
- A perpendicular from centre bisects the chord into two equal parts.



- Only one tangent can pass through a point on the circle



- The angle subtended by an arc at the centre of a circle is **twice** the angle subtended at any point on the circumference

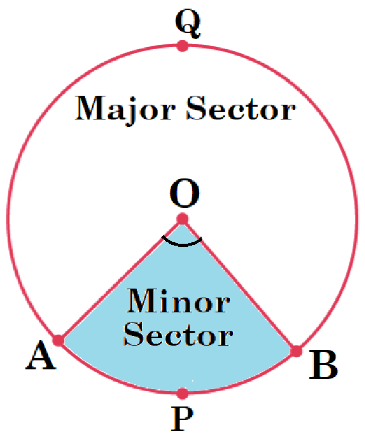


That is, $\angle 2 = 2\angle 1$

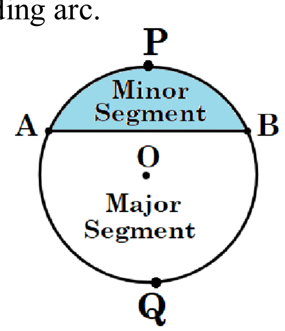
MIND MAPS
Class X - Maths

AREAS RELATED TO CIRCLES

Sector of a circle
It is that portion or part of the circle which is **enclosed by two radii** and the **corresponding arc**.



Segment of a circle
It is that portion of the circle which is enclosed between a chord and the corresponding arc.



In a clock, the
- Angle made by minute-hand in 1 minute $= 6^\circ$
- Angle made by hour-hand in 1 minute $= 0.5^\circ$

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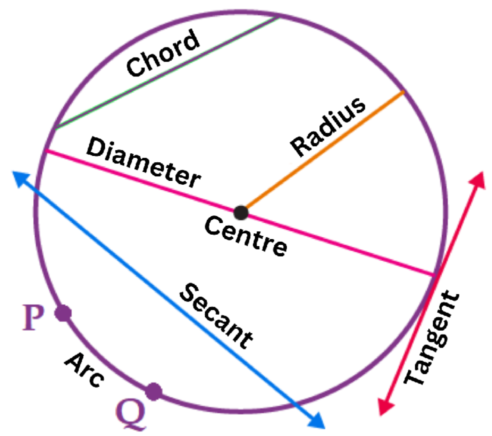
Basic introduction
- A **circle** is the set of all points in a plane at a fixed distance from a fixed point
- Fixed point → **Centre**
- Fixed distance → **Radius**
- **Radius (r)** : Distance from centre to any point on the circle
- **Diameter (2r)** : Longest chord of the circle, passes through centre
- **Chord** : Line segment joining any two points on the circle
- **Arc** : Part of the circumference (PQ)
- **Secant** : A line intersecting the circle at **two points**
- **Tangent** : A line which touches the circle at **exactly one point**, this point is called **Point of contact**
- **Semi-circle :** Diameter of circle divides it into two equal parts (or, two equal arcs). Each part is called a semi-circle



Common mistakes to avoid
- Confusing sector and segment
- Forgetting to subtract triangle area in segment
- Missing units (cm^2, m^2)
- Forgetting to put correct value of π, as directed

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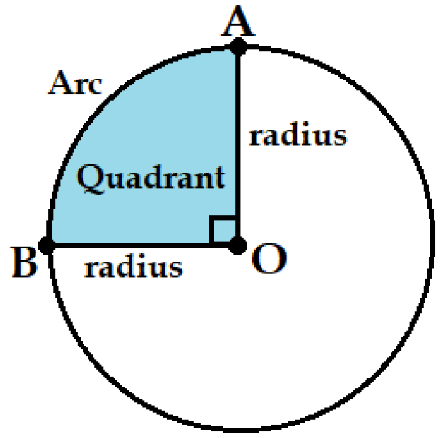
Quadrant of a circle : One fourth part of a circle is called the quadrant of the circle



Circumference of a circle : The distance covered by travelling once around a circle is its perimeter, which is usually called its circumference.
That is, Circumference (arc length of semicircle of radius r) is given by $C = 2\pi r$.
Further, note that **Perimeter of semi-circle** of radius $r = \pi r + 2r = \pi r + d$, where d is the diameter.

Some important formulae

a) Area of circle $= \pi r^2$

b) Area of the sector of angle $\theta = \dfrac{\theta}{360} \times \pi r^2$

c) Length of the arc of a sector of angle $\theta = \dfrac{\theta}{360} \times 2\pi r$

d) Area of the segment of a circle
= Area of the corresponding sector
− Area of corresponding triangle

$$= \frac{\theta}{360} \times \pi r^2 - r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

e) Perimeter of equilateral triangle $= 3a$

f) Area of equilateral triangle $= \dfrac{\sqrt{3}}{4} a^2$

g) Area of rhombus $= \dfrac{1}{2}$(Product of diagonals)

h) Area of Parallelogram = Base × Corresponding height

i) Area of trapezium

$$= \frac{1}{2}\left[(\text{Sum of parallel sides}) \times \left(\begin{matrix}\text{Distance between} \\ \text{parallel sides}\end{matrix}\right)\right]$$

SURFACE AREAS & VOLUMES

MIND MAPS
Class X - Maths

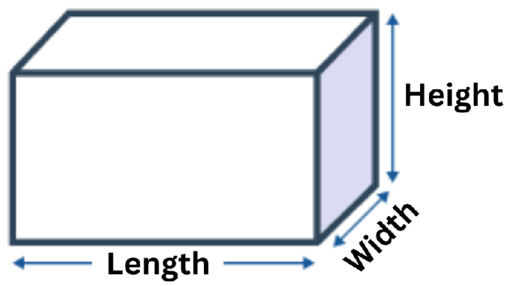
Surface areas of solid bodies

Surface area of a solid body is the area of all of its surfaces together. It is measured in square units.

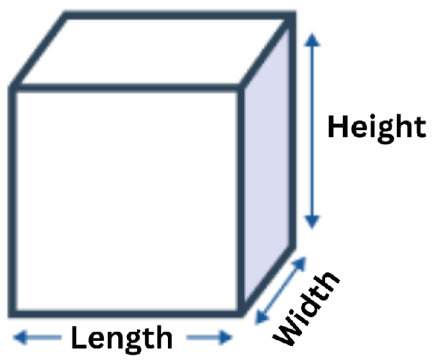
Volume of solid bodies

Volume of a solid body is the space occupied by it. It is measured in cubic units.

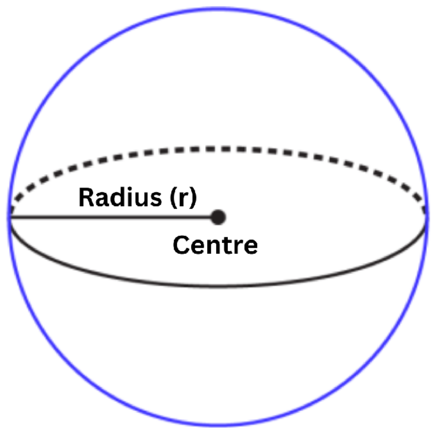
Some important solid bodies



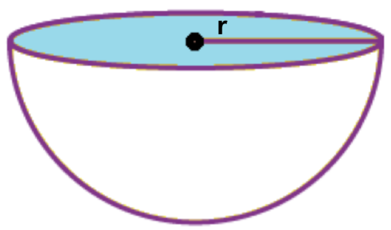
Cuboid



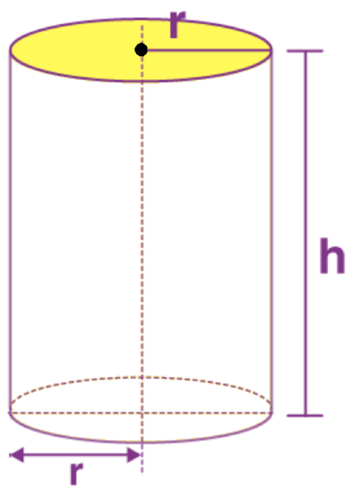
Cube



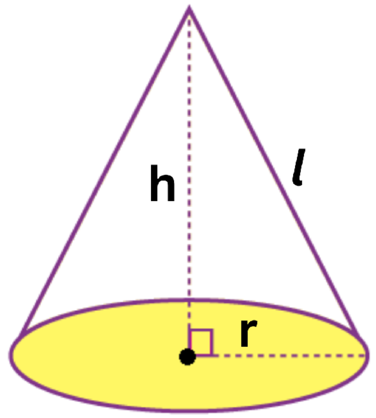
Sphere



Hemisphere



Cylinder



Cone

Some important formulae

Cuboid

Lateral surface area $= 2(l+b)h$

Total surface area $= 2(lb+bh+hl)$

Volume $= lbh$

Cube

Lateral surface area $= 4a^2$

Total surface area $= 6a^2$

Volume $= a^3$

Sphere

Surface area $= 4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$

Hemisphere

Curved surface area $= 2\pi r^2$

Surface area $= 3\pi r^2$

Volume $= \frac{2}{3}\pi r^3$

Cylinder

Curved surface area $= 2\pi r h$

Total surface area $= 2\pi r^2 + 2\pi r h$

Volume $= \pi r^2 h$

Cone

Curved surface area $= \pi r l$, where $l^2 = r^2 + h^2$

Total surface area $= \pi r^2 + \pi r l$

Volume $= \frac{1}{3}\pi r^2 h$

Important exam tips

- Avoid confusion between CSA and TSA
- Use slant height (l) for CSA of cone
- Avoid unit mismatch
- Make diagram while solving the problems, as it improves presentation
- Use the value of $\pi = 3.14$ or $\pi = \frac{22}{7}$, as directed

MIND MAPS
Class X - Maths

STATISTICS

Mean of Grouped Data

In general, it is obtained by dividing the sum of all observations by the total number of observations. It is denoted by $\bar{x}$.

(a) Direct method : $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

where x_i represents the variates (class-marks) and f_i their corresponding frequencies.

(b) Assumed mean method : $\bar{x} = a + \dfrac{\sum f_i d_i}{\sum f_i}$

where 'a' is the **assumed mean** and $d_i = x_i - a$ is the deviation of 'a' from each of the variates x_i.

(c) Step deviation method : $\bar{x} = a + \dfrac{\sum f_i u_i}{\sum f_i} \times h$

where 'a' is the **assumed mean** and 'h' is the class-width. Also $u_i = \dfrac{x_i - a}{h}$.

◈ Assumed mean and step deviation methods are known as **Short-cut method** also.

Median

It is the middle most or the central observation, when the given set of observations are arranged in an order.

- **Cumulative frequency**

The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class.

- **Median class**

A class having cumulative frequency just greater than half the sum of all the frequencies (popularly termed as the $\dfrac{N}{2}$ value) is called the median class. In other words, the median class is obtained by marking that class interval which corresponds to the cumulative frequency which is just greater than the $\dfrac{N}{2}$ value.

◈ Median of Discrete Frequency Distribution

STEP 1 Find the cumulative frequency.

STEP 2 Find $N = \sum f_i$ and, then $\dfrac{N}{2}$.

STEP 3 Observe the cumulative frequency which is **just greater than the $\dfrac{N}{2}$ value** and find out the corresponding value of the variable x.

STEP 4 The value of the variable obtained in the **STEP 3** is the required Median.

◈ Median of Grouped Data or Continuous Frequency Distribution

- $\text{Median} = l + \left(\dfrac{\frac{N}{2} - C}{f} \right) \times h$

where $l =$ lower limit of the median-class, $N = \sum f_i =$ number of observations, $C =$ cumulative frequency of the class preceding the median class, $f =$ frequency of the median-class, $h =$ class-size.

Mode of Grouped Data

The value of observation having maximum frequency is the mode of the given data.

- $\text{Mode} = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

where $l =$ lower limit of the modal-class,
$h =$ size of the class interval,
$f_1 =$ frequency of the modal-class,
$f_0 =$ frequency of the class preceding the modal-class,
$f_2 =$ frequency of the class succeeding the modal-class.

Note : The **modal-class** is that class which has got the maximum frequency.

Empirical relationship between the three measures of Central Tendency

$$3\,\text{Median} = \text{Mode} + 2\,\text{Mean}$$

MIND MAPS
Class X - Maths

PROBABILITY

Description of the Playing Cards

- Total no. of playing cards : 52
- Total no. of Red cards : 26
- Total no. of Black cards : 26 (so, 26 + 26 = 52)
- No. of Ace cards : 4 (2 Black and 2 Red)
- Three Face cards are : Jack, Queen, King
- No. of Face cards : 4 Jacks, 4 Queens, 4 Kings (so, total face cards : $4 \times 3 = 12$)
- No. of Red coloured Face cards : 6 (Two from each of the faces)
- No. of Black coloured Face cards) : 6 (Two from each of the faces)
- Four Suits are : Club, Spade (Both black); Heart, Diamond (Both red)
- No. of cards in each suit : 13 (so, $13 \times 4 = 52$)
- **Numbering on the cards** is done from 2 to 10. Sometimes Ace is numbered as 1 but if so, then it will be mentioned in the question. Thus we have four cards with each numbers from 2 to 10 from each of the four suits.

Important terms

Experiment
It is an activity which results in some well-defined outcomes (results).

Trial
It is an activity performed once which results in one or several outcomes.

Random experiment
It is an experiment with **uncertain outcome**. That is, an experiment in which all the possible outcomes are known in advance but exactness of outcome cannot be predicted.

Elementary event
An outcome of a random experiment is called an elementary event.

Occurrence of an event
An event E associated to a random experiment is said to occur if any one of the elementary events associated to the event E is an outcome.

Non-occurrence of an event
An event E associated to a random experiment is said to be non-occurring if there is no outcome associated to the event E.

Favourable elementary event
An elementary event E is said to be favourable if it is as per the definition of the event.

Impossible event
An event is said to be impossible if it can never happen. The probability of an impossible event is 0 *i.e.*, zero.

Sure event
If an event is bound to happen, then it is a sure event. The sure event is also called as **Certain event**. The probability of an impossible event is 1.

Sample space
It is the collection (i.e., set) of all the possible outcomes in an experiment. Usually it is symbolized by the upper case letter S. And the elements of the set is put into brackets {...}.

- Total number of outcomes in n throws of a coin $= 2^n$
- Total number of outcomes in n throws of a die $= 6^n$

Common mistakes to avoid

- Confusing sector and segment
- Ignoring **'favourable'** vs **'total'** outcomes logic
- Not simplifying final answers into fractions
- Misinterpreting **'at least'** or **'not'** type questions

Probability of an event

$$P(E) = \frac{\text{Number of outcomes favourable to event E}}{\text{Number of all possible outcomes of the experiment}}$$

Important properties

- $0 \le P(E) \le 1$
- $P(E) + P(\text{not } E) = 1$ i.e., $P(E) + P(\bar{E}) = 1$

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SOLVE SMARTER.

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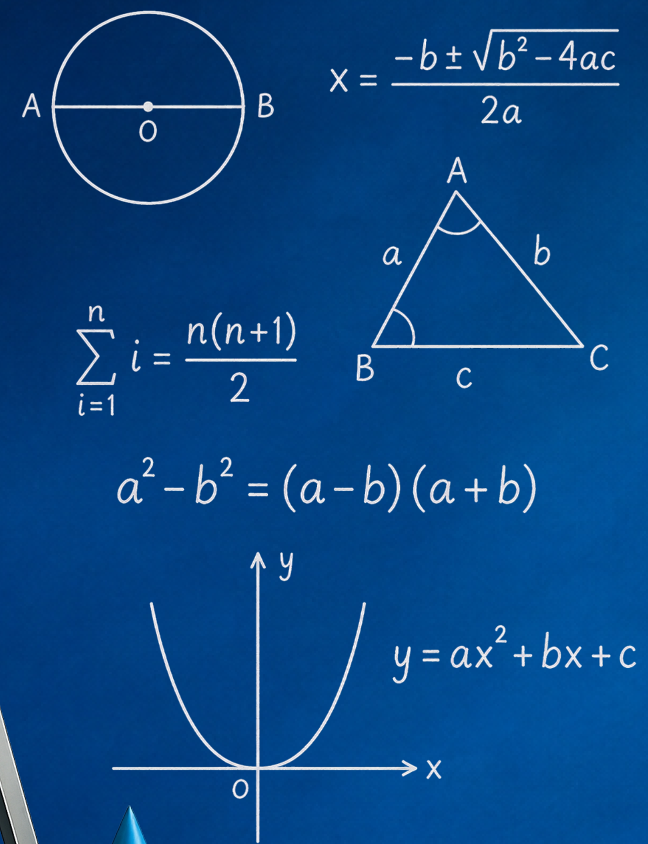

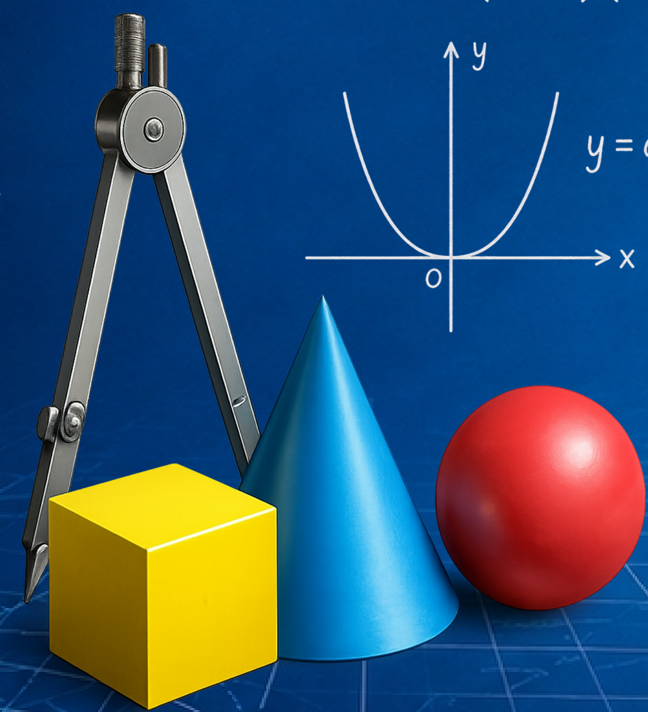

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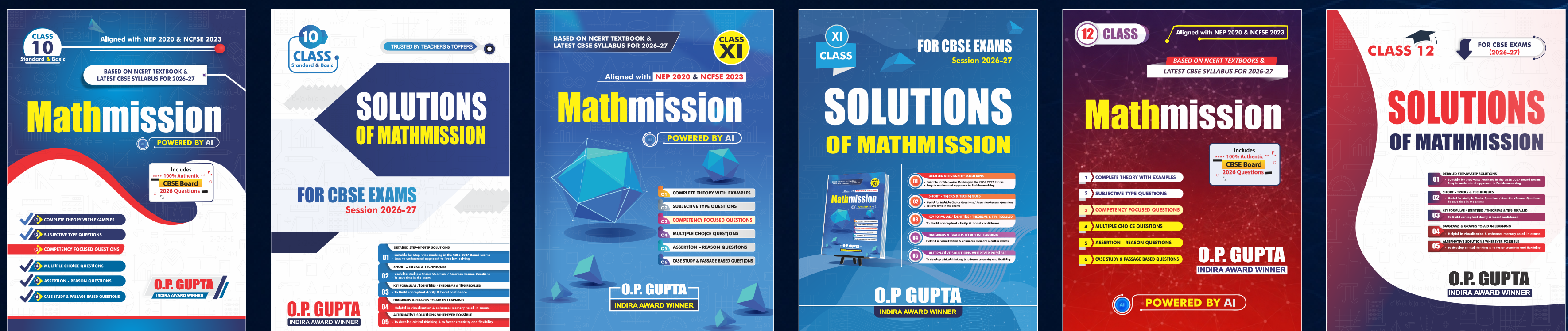

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